Fisher’s Exact Test

STA 610 - Applied Statistics for Health Professions

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## Learning Objectives

* State hypotheses for Fisher’s exact test
* State and check assumptions of Fisher’s exact test
* Obtain and interpret results of Fisher’s exact test

Let’s begin by loading some R packages for this activity using the code below. Note: if it is the first time you are using an R package, you may need to install it first using the install.packages() function.

# Load necessary packages  
library(tidyverse)  
library(ggthemes)  
library(flextable)  
library(janitor)  
library(broom)

Next, we can set default theme settings for plots, and load some functions to simplify table customization and creation using the code below.

# Set ggplot theme for visualizations  
theme\_set(ggthemes::theme\_few())  
  
# Set options for flextables  
set\_flextable\_defaults(na\_str = "NA")  
  
# Load function for printing tables nicely  
source("https://raw.githubusercontent.com/dilernia/STA323/main/Functions/make\_flex.R")

## Fisher’s exact test

Fisher’s exact test is a hypothesis test for testing if there is an association between two categorical variables, addressing the same question as the -test of independence. However, the -test of independence is based on an asymptotic approximation, meaning that its accuracy is dependent on the sample size being sufficiently large, whereas Fisher’s exact test is valid even for small sample sizes.

### Hypotheses

Hypotheses for Fisher’s exact test are identical to that of the -test of independence:

These hypotheses should be tailored to provide the context for a given problem by giving explicit descriptions of X and Y.

### Assumptions

The assumptions for Fisher’s exact test are are less restrictive than the -test of independence.

* Independent observations (commonly violated when observations consist of repeated measurements across time)
* Both variables being studied must be categorical

### The Lady Tasting Tea

The motivation for Fisher’s exact test is a part of statistical lore. Supposedly, Dr. Muriel Bristol, a researcher at Rothamsted Research station with Ronald Fisher, claimed that she could distinguish whether tea 🍵 was poured into a cup before or after adding milk 🐄 🥛 just by tasting it. Fisher was skeptical of this claim. To test it he designed an experiment where several cups of tea were randomly prepared out of sight of Dr. Bristol with either having milk added before tea or milk added after. Dr. Bristol then tasted all prepared cups of tea and indicated whether she believed milk was added first or not. This experiment motivated Fisher’s exact test as a way to test Dr. Bristol’s claim.

We will emulate the setting of “The Lady Tasting Tea” with an experiment of our own. Instead of milk and tea though, we will use store-brand and Chips Ahoy brand cookies 🍪. Everyone will leave the room allowing me to randomly place either a store-brand or Chips Ahoy cookie at your desk. You will then return and individually decide if you believe your cookie is either store-brand or Chips Ahoy brand. Then, you will record your answer by responding to this [Google Form](https://forms.gle/3Q6BprfSuU7kjw9v6). Using this data, we will investigate whether we as a class are able to determine if a cookie is Chips Ahoy brand or store-brand using Fisher’s exact test. This will be considered a single-blind experiment since you, the participants, are not aware which treatment / cookie type you were given, but as the person conducting the study, I do know the true cookie brands.

Please be aware that the cookies provided for this in-class activity may contain common allergens such as nuts, dairy, and gluten. If you have any allergies or dietary restrictions, exercise caution and know that you do not have to eat these if you have allergy restrictions or simply do not want to.

➡️ What are the response and explanatory variables in this scenario?

➡️ Formally state the hypotheses for our question of interest.

Next,

➡️ Import the [data](https://github.com/dilernia/STA323/blob/main/Data/cookies.csv) from our experiment into R using the code below.

# Importing data from GitHub  
cookies <- readr::read\_csv("https://raw.githubusercontent.com/dilernia/STA323/main/Data/cookies.csv")

Let’s obtain a contingency table to view the results of our study.

# Creating contingency table  
contingencyTable <- cookies %>%   
 janitor::tabyl(var1 = Truth,   
 var2 = Guess)  
  
# Printing table  
contingencyTable %>%   
 make\_flex(caption = "Observed counts for true brand and guessed brand of cookies", ndigits = 0)

We can create a clustered bar chart to visualize the same information conveyed in the contingency table.

# Creating a clustered bar chart  
cookies %>%   
 dplyr::count(Truth, Guess, .drop = FALSE) %>%   
 dplyr::filter(!is.na(Truth), !is.na(Guess)) %>%   
 mutate(Truth = fct\_reorder(Truth, n)) %>%   
 ggplot(aes(x = Truth, y = n,  
 fill = Guess)) +   
 geom\_col(position="dodge", color = "black") +  
 scale\_fill\_few() +  
 scale\_y\_continuous(expand = expansion(mult = c(0, 0.10))) +  
 labs(title = "True and guessed cookies brands",  
 y = "Guessed brand",  
 x = "True cookie brand",  
 fill = "Guessed brand")

We can also create a dumbbell plot to show this information as well.

# Creating dumbbell chart  
cookie\_counts <- cookies %>%   
 dplyr::count(Guess, Truth, .drop = FALSE) %>%   
 dplyr::filter(!is.na(Guess), !is.na(Truth))   
  
cookie\_counts %>%   
 ggplot(aes(x = n, y = Truth,  
 color = Guess, fill = Guess)) +   
 geom\_line(aes(group = Truth), color = "black") +  
 geom\_point(pch = 21, color = "black", size = 5) +  
 scale\_fill\_few() +  
 labs(title = "True and guessed cookies brands",  
 x = "Frequency",  
 y = "True cookie brand",  
 fill = "Guessed brand") +  
 theme(legend.position = "bottom",  
 strip.background.y = element\_rect(linetype = "solid", color = "black"))

➡️ Based on the contingency table and visualizations, do we feel that we as a class are able to determine if a cookie is Chips Ahoy brand or store-brand?

➡️ Implement Fisher’s exact test to formally test whether or not the true brand and guessed brand of the cookies are related.

# Implementing Fisher's exact test  
fisherRes <- fisher.test(contingencyTable)  
  
# Printing model output  
fisherRes %>%   
 broom::tidy() %>%   
 make\_flex(caption = "Results of Fisher's exact test",  
 ndigits = 2)

➡️ State our p-value, decision, and conclusion in the context of the problem testing at the significance level, citing specific evidence from the obtained output.

➡️ Based on Fisher’s exact test, were we able to successfully distinguish between store-brand and Chips Ahoy brand cookies? Why or why not?

For the -test of independence, there are slightly more assumptions.

* Independent observations (commonly violated when observations consist of repeated measurements across time)
* Both variables being studied must be categorical
* All expected counts under the null hypothesis should be at least 5

➡️ Implement the analogous -test of independence. Regardless of whether or not assumptions are met, provide the hypotheses, test statistic, p-value, decision, and interpretation of the results in context.

➡️ Do the results align with Fisher’s exact test? How similar are the p-values for the two tests?

# Implementing a chi-square test of independence  
chi2Res <- chisq.test(contingencyTable, tabyl\_results = TRUE)  
  
# Printing model output  
chi2Res %>%   
 broom::tidy() %>%   
 make\_flex(caption = "Results of the chi-square test of independence",  
 ndigits = 2)